Gauge Symmetry Breakdown due to Dynamical Higgs Scalar [a]

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Assuming dynamical spontaneous breakdown of chiral symmetry for massless gauge theory without scalar fields, we present a method how to construct an effective action of the dynamical Nambu-Goldstone bosons and elemetary fermions by using auxiliary fields. Here dynamical particles are asssumed to be composed of elementary fermions. Various quantities including decay constants are calculated from this effective action. This technique is also applied to gauge symmetry breakdown, $SU(5) \rightarrow SU(4)$, to obtain massive gauge fields.

1 Introduction

1.1 Massless $U(1)$ Gauge Theory

Bound states composed of fermion-anti-fermion pairs can often be expressed as bilocal fields. Those fields are so far treated classically using the Bethe-Salpeter equation. Or only its vacuum expectation vaule is caluculated since it is also a c-number. Here in this report we present a method [1] how to treat those bilocal fields as local ones so that one can use Feynman diagrams to calculate physical quantities. That is, we propose a method by which one can treat bilocal fields as quantum ones at least in the first order of approximation.

We use massless $U(1)$ gauge theory to descibe our idea as the simplest example. Starting from the following Lagrangian,

$$\mathcal{L}_0 = -\frac{1}{4}\left(F^{\mu\,\nu}\right)^2 - \frac{1}{2\alpha}\left(\partial_\mu A^\mu\right)^2 + \bar{\psi}\, i\not{D}\,\psi, \tag{1}$$

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where $D_\mu = \partial_\mu - igA_\mu$ and α is a gauge parameter. We just describe how to obtain the effective Lagrangian below. First functionally integrate patition function using Eq.(9) over gauge fields. Then the resultant four-fermi interactions are cancelled by introducing bilinear terms of auxiliary fields [2]. Finally we are left with the effective Lagrangian including fermions as well as auxiliary fields which are decomposed into local fields as

$$S_{\rm eff} = \int d^4x\ \bar{\psi}(x)\ i\partial\!\!\!/\, \psi(x) - \int d^4x\, d^4y\ \bar{\psi}(x)\Big[\phi(x,y) + i\gamma_5\pi(x,y)\Big]\psi(y)$$
$$+ \frac{1}{2}\int d^4x\, d^4y\ \Big[\phi(x,y)D^{-1}(x-y)\phi(y,x) + \pi(x,y)D^{-1}(x-y)\pi(y,x)\Big] + \ldots \quad (2)$$

with

$$D(x-y) = \frac{g^2}{4}\, g^{\mu\,\nu} D_{\mu\,\nu}(x-y) = -\frac{\lambda}{(x-y)^2}, \quad (3)$$
$$\phi(x,y) = \frac{\phi_0(x-y)}{f}\left[f + \sigma\left(\frac{x+y}{2}\right)\right], \quad (4)$$
$$\pi(x,y) = \frac{\phi_0(x-y)}{f}\ \varphi\left(\frac{x+y}{2}\right). \quad (5)$$

and a gauge boson propagator $D_{\mu\,\nu}$, $D^{-1} = 1/D$, and $\lambda = (3+\alpha)g^2/(16\pi^2)$. Eqs. (4, 5) are the key point of our paper. This decomposition of these fields, $\phi(x,y)$ and $\pi(x,y)$ can be interpreted as a decomposition of internal degrees of freedom depending on $x-y$, i.e., $\phi_0(x-y)$, and a total degree of freedom depending on $(x+y)/2$, i.e., $\sigma((x+y)/2)$ and $\varphi((x+y)/2)$ which are treated as local fields. Translational invariant quntity $\phi_0(x-y)$ is regarded as a vacuum expectation value of a bilocal field $\phi(x,y)$. This intuitive interpretation can also be supported by chekcing that mass of the Nambu-Golston (NG) boson vanishes by calculating only bilinear terms in φ which consist of tree term and fermion one-loop diagram from Eq.(2). The coefficients of these two bilocal fields become the same, $\phi_0(x-y)$, can be derived by this vanishing NG boson mass. This $\phi_0(x-y)$ satisfies the gap equation, namely it gives a fermion mass $\Sigma(q)$ as

$$\partial_q^2\Sigma(q) + \frac{4i\lambda\Sigma(q)}{q^2 - \Sigma^2(q)} = 0, \quad (6)$$

with $\Sigma(q) = \int d^4r/(2\pi)^4\, \phi_0(r)\exp(-iqr)$.

1.2 Decay Constant in $U(1)$ Gauge Theory

Using this result, the physical quntity to be calculated first is a decay constant in the $U(1)$ case defined by

$$\left\langle 0 \left| \bar{\psi}(0)\, \gamma_\mu \gamma_5 \psi(0) \right| \varphi(q) \right\rangle = -2i\, f\, q_\mu . \tag{7}$$

The Bethe-Salpeter equation is used to describe the left hand side of Eq. (7).[3] Our method tells us to calculate just one fermion loop which connects an axial vector vertex and the NG boson, $\varphi(q)$. Our method reproduces the so-called Pagels-Stokar[4] formula for a decay constant f,

$$f^2 = \frac{1}{2\,(2\pi)^2} \int x dx\, \frac{\Sigma(x)\Big[\Sigma(x) - x\Sigma'(x)/2\Big]}{\Big[x + \Sigma^2(x)\Big]^2}, \tag{8}$$

where momentum space is converted into Eucledean space, i.e., $x = -p^2$.

2 Dynamical Breakdown of Massless $SU(5)$ Gauge Theory

The next application of our method is to calculate gauge boson masses when gauge symmetry dynamically breaks down. We study the case in which massless $SU(5)$ gauge theory dynamically breaks down to $SU(4)$ which was predicted as the most favorable breaking pattern due to a tumbling scenario.[5] However nobody has ever shown how to calculate gauge boson masses when symmetry breaks down. Peole have just predicted that masses may be obtained by applying the Pagels-Stokar formula and multiplying a gauge coupling constant since there is no way to calculate those qunatities so far. We will show our method can do this job.

The difference between this section and Subsections 1.1 and 1.2 is that whether there is a real gauge field which couples to a fermion axial vector vertex or not. In the former section we do not have such a coupling, while in this gauge theory we do.

Now we start from the following Lagrangian

$$\begin{aligned}\mathcal{L}_0 \,=\, & -\frac{1}{4}\left(F^A_{\mu\,\nu}\right)^2 + \, i\left(\overline{\psi}\right)^i \left[\partial\!\!\!/\, \delta_i{}^j - ig A\!\!\!/^A \left(T^A\right)_i{}^j\right] \psi_j \\ & + \frac{i}{2}(\overline{\chi})^{\;i\,j} \Big[\partial\!\!\!/\, \delta_j{}^k - 2ig A\!\!\!/^A \big(T^A\big)_j{}^k\Big]\, \chi_{k\,i},\end{aligned} \tag{9}$$

where $A = 1 \sim 24$, $(\psi^c)^i \sim \underline{5}^*$ and $\chi_{i\,j} \sim \underline{10}$ left handed fermions are introduced. Our breaking pattern tells us that 5-dimensional $\phi_i \sim \underline{10} \times \underline{10}$ complex

scalar bound state plays a role of Higgs scalar, which should give mass to $\underline{1}$ and $\underline{4}/\underline{4}^*$ representations out of $\underline{24} = \underline{15} + \underline{4} + \underline{4}^* + \underline{1}$ representations under $SU(4)$. $\underline{15}$ corresponds to massless $SU(4)$ gauge fields which are functionally integrated out and give four-fermi terms that are cancelled by bilinear terms in the bilocal auxiliary fields.

Corresponding to Eqs. (4, 5), we have

$$\begin{aligned}\phi_i(x,y) &= \left\{ \frac{\phi_0(x-y)}{v} \left[v + \sigma\left(\frac{x+y}{2}\right)\right] \exp\left[\frac{i\pi^\alpha((x+y)/2)T^\alpha}{v}\right] \eta \right\}_i \qquad (10)\\ &= \frac{\phi_0(x-y)}{v}\left[v+\sigma\left(\frac{x+y}{2}\right)\right]\delta_i{}^5 + i\frac{\phi_0(x-y)}{v}\pi^\alpha\left(\frac{x+y}{2}\right)(T^\alpha)_i{}^5 + \ldots,\\ \eta_i &\equiv \delta_i{}^5. \qquad (11)\end{aligned}$$

Using this expression, we can calculate gauge boson masses, $\underline{1}$ and $\underline{4}$. However these are related to $\underline{6}$ fermion mass $\Sigma(q) = -2\int d^4r/(2\pi)^4\phi_0(r)\exp(-iqr)$ which is the only fermion mass available. Fermion mass is only obtained after $SU(5)$ breaks down to $SU(4)$, which means mass ratio between $\underline{1}$ and $\underline{4}$ gauge bosons is not given by that as expected from an elementary Higgs. In a sense, gauge boson masses are available after gauge symmetry breaks down due to fermion chiral condensation occurs. All these details will be given in a separate paper. [6]

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